

Modern Physics Letters A
 © World Scientific Publishing Company

5D Solutions to Λ CDM Universe Derived from Global Brane Model

Yongli Ping, Lixin Xu, Baorong Chang, Molin Liu and Hongya Liu

*School of Physics and Optoelectronic Technology,
 Dalian University of Technology, Dalian, Liaoning 116024, P.R.China
 ylping@student.dlut.edu.cn*

Received
 Revised

An exact solution of brane universe is studied and the result indicates that Friedmann equations on the brane are modified with an extra term. This term can play the role of dark energy and make the universe accelerate. In order to derive the Λ CDM Universe from this global brane model, the new solutions are obtained to describe the 5D manifold.

Keywords: Λ CDM; brane; cosmology.

PACS numbers: 04.50.+h, 98.80.-k, 02.40.-k

1. Introduction

Recent observations indicate that our universe is undergoing accelerated expansion^{1,2} and dominated by a negative pressure component dubbed dark energy. Obviously, a natural candidate to dark energy is a cosmological constant with equation of state $w_\Lambda = -1$. Einstein (1917) introduced the cosmological constant Λ , because he believed that the universe is static.³ However, Friedmann (1922) discovered an expanding solution to the Einstein field equations in the absence of Λ and Hubble (1929) found the universe was expanding.^{4,5} Soon after, Einstein discarded the cosmological constant and admitted his greatest blunder. Although abandoned by Einstein, the cosmological constant staged several come-backs. It was soon realized that, since the static Einstein universe is unstable to small perturbations, one could construct expanding universe models which had a quasi-static origin in the past, thus ameliorating the initial singularity which plagues expanding FRW models. Theoretical interest in Λ remained on the increase during the 1970s and early 1980s with the construction of inflationary models, in which matter (in the form of a false vacuum, as vacuum polarization or as a minimally coupled scalar-field) behaved precisely like a weakly time-dependent Λ -term. The cosmological constant makes an important appearance in models with spontaneous symmetry breaking.⁶ The current interest in Λ stems mainly comes from observations of Type Ia high redshift supernovae which indicate that the universe is accelerating expansion fueled perhaps by a small cosmological Λ -term.^{1,2} The review about the cosmological

2 *Y. Ping et al.*

constant can be seen.^{6,7,8,9}

It is proposed that our universe is a 3-brane embedded in a higher-dimensional space.^{10,11,12,13,14,15,16,17} In brane-world model, gravity can freely propagate in all dimensions, while standard matter particles and forces are confined on the 3-brane. A five-dimensional ($5D$) cosmological model and derived Friedmann equations on the branes are considered by Binetruy, Deffayet and Langlois (BDL),¹⁸ for a recent review, it can be seen.^{19,20} Brane-world models of dark energy are studied²¹ and accelerating universe comes from gravity leaking to extra dimension in DGP brane.²²

In this paper, we derive Λ CDM universe from global brane model with a Ricci-flat bulk characterized by a class of exact solutions. The solutions were firstly presented by Liu and Mashhoon and restudied latter by Liu and Wesson.^{23,24} And these solutions are algebraically rich because they contain two arbitrary functions of time t . The solutions are utilized in cosmology^{25,26,27,28,29,30,31,32,33} and are relate to the brane model.^{34,35,36} In order to induce Λ CDM universe from global brane model, more exact solutions of the $5D$ bulk are obtained.

2. Friedmann equations in global brane universes

A class of $5D$ Ricci-flat cosmological solution reads²³

$$dS^2 = B^2 dt^2 - A^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) - dy^2, \quad (1)$$

$$A^2 = (\mu^2 + k) y^2 + 2\nu y + \frac{\nu^2 + K}{\mu^2 + k}, \quad (2)$$

$$B = \frac{1}{\mu} \frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\mu}, \quad (3)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\psi^2$; $\mu = \mu(t)$ and $\nu = \nu(t)$ are two arbitrary functions of time t ; k is the 3D curvature index ($k = \pm 1, 0$), and K is a constant. Because the $5D$ manifold (1)-(3) is Ricci-flat, we have $I_1 \equiv R = 0$, $I_2 \equiv R^{AB} R_{AB} = 0$, and

$$I_3 \equiv R^{ABCD} R_{ABCD} = \frac{72K^2}{A^8}, \quad (4)$$

so K is related to the $5D$ curvature. They are used as the bulk solutions of the BDL-type brane model. To obtain brane models for using the Z_2 reflection symmetry on A and B , they are set as³⁶

$$\begin{aligned} A^2 &= (\mu^2 + k) y^2 - 2\nu |y| + \frac{\nu^2 + K}{\mu^2 + k}, \\ B &= \frac{1}{\mu} \frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\mu}. \end{aligned} \quad (5)$$

Then the corresponding $5D$ bulk Einstein equations are taken as

$$G_{AB} = \kappa_{(5)}^2 T_{AB},$$

$$T_B^A = \delta(y) \text{diag}(\rho_1, -p_1, -p_1, -p_1, 0) + \delta(y - y_2) \text{diag}(\rho_2, -p_2, -p_2, -p_2, 0) \quad (6)$$

where the first brane is at $y = y_1 = 0$ and the second is at $y = y_2 > 0$. In the bulk $T_{AB} = 0$ and $G_{AB} = 0$, Eq.(6) are satisfied by (5). On the branes, Liu had solved Eq.(6) in Ref. 36. We adopt the result at $y = y_1 = 0$ and $y = y_2 > 0$ as follows:

$$\kappa_{(5)}^2 \rho_1 = \frac{6\nu}{A_1^2}, \quad (7)$$

$$\kappa_{(5)}^2 p_1 = -\frac{2}{\dot{A}_1} \frac{\partial}{\partial t} \left(\frac{\nu}{A_1} \right) - \frac{4\nu}{A_1^2}, \quad (8)$$

and

$$\kappa_{(5)}^2 \rho_2 = \frac{6}{A_2} \left(\frac{\mu^2 + k}{A_2} y_2 - \frac{\nu}{A_2} \right), \quad (9)$$

$$\begin{aligned} \kappa_{(5)}^2 p_2 = & -\frac{2}{\dot{A}_2} \frac{\partial}{\partial t} \left(\frac{\mu^2 + k}{A_2} y_2 - \frac{\nu}{A_2} \right) \\ & - \frac{4}{A_2} \left(\frac{\mu^2 + k}{A_2} y_2 - \frac{\nu}{A_2} \right), \end{aligned} \quad (10)$$

where, A_1 is the scale factor on $y = y_1 = 0$ brane and A_2 is the scale factor on $y = y_2 > 0$ brane.

Now, we consider the universe on the second brane, i.e. $y = y_2 > 0$. From the 5D metric (1), the Hubble and deceleration parameters on the $y = y_2$ brane can be defined as

$$H_2(t, y) \equiv \frac{1}{B_2} \frac{\dot{A}_2}{A_2} = \frac{\mu}{A_2}, \quad (11)$$

$$q_2(t, y) = -\frac{A_2 \dot{\mu}}{\mu \dot{A}_2}. \quad (12)$$

Substituting Eq.(11) into Eq.(9) to eliminate μ^2 , Eq. (9) can be rewritten into a new form as

$$H_2^2 + \frac{k}{A_2^2} = \frac{\kappa_{(5)}^2}{6y_2} \left(\rho_2 + \frac{6}{\kappa_{(5)}^2} \frac{\nu}{A_2^2} \right). \quad (13)$$

We define $\rho_x = \frac{6}{\kappa_{(5)}^2} \frac{\nu}{A_2^2}$ and it can play the role of dark energy. Then from the Eq.(10), we have

$$\frac{2\mu \dot{\mu}}{A_2 \dot{A}_2} + \frac{\mu^2 + k}{A_2^2} = -\frac{\kappa_{(5)}^2}{2y_2} (p_2 - p_x), \quad (14)$$

where $p_x = -\frac{2}{\kappa_{(5)}^2} \left(\frac{\dot{\nu}}{A_2 \dot{A}_2} + \frac{\nu}{A_2^2} \right)$. Meanwhile, the conservation law $T_{A;B}^B = 0$ gives

$$\dot{\rho}_2 + 3(\rho_2 + p_2) \frac{\dot{A}_2}{A_2} = 0. \quad (15)$$

From Eqs. (13) and (14), it can be seen that the extra term which will be treated as dark energy have been induced on the brane.³⁷ By assuming that only dark matter is contained on the brane, we have $p_2 = 0$ and $\rho_2 = \rho_{20} A_{20}^3 A_2^3$. Then, from Eq. (13) and Eq. (14), for $k = 0$, EOS of dark energy, dimensionless density parameters and deceleration parameters q_2 with $A_{20} = 1$ and $\nu_0 = 1$ can be obtained

$$w_x = \frac{p_x}{\rho_x} = -\frac{1}{3} \left(\frac{A_2 \dot{\nu}}{A_2 \nu} + 1 \right), \quad (16)$$

$$\Omega_2 = \frac{1}{1 + \nu A_2 (1 - \Omega_{20})}, \quad (17)$$

$$\Omega_x = 1 - \Omega_2, \quad (18)$$

$$q_2 = \frac{1}{2} \left[- \left(\frac{\dot{\nu}}{A_2 \dot{A}_2} + \frac{\nu}{A_2^2} \right) \frac{1 - \Omega_{20}}{1 + \nu A_2 (1 - \Omega_{20})} + 1 \right], \quad (19)$$

where Ω_{20} is current value of matter density parameter Ω_2 . If the function ν is given, the evolutions of all cosmic observable parameters in (16)-(19) are determined uniquely.

3. Solutions to Λ CDM universe derived from global brane

The Friedmann equations in four-dimensional Λ CDM universe are

$$H^2 + \frac{k}{a^2} = \frac{\kappa_{(4)}^2}{3} \rho + \frac{\Lambda}{3}, \quad (20)$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa_{(4)}^2}{6} (\rho + 3p) + \frac{\Lambda}{3}, \quad (21)$$

where there is only the matter i.e $\rho = \rho_m$ and $p = p_m = 0$. Meanwhile, the equation of state on the cosmological constant Λ is $w_\Lambda = -1$. In order to get the LCDM universe from the global brane, from Eq. (16), we find when

$$p_x = -\rho_x, \quad (22)$$

the property on the brane tends to Λ CDM universe. And, we can find $\kappa_{(4)}^2 = \kappa_{(5)}^2 / (2y_2)$. Since $\kappa_{(5)}^2 = M_{(5)}^{-3}$ and $\kappa_{(4)}^2 = M_{(4)}^{-2}$, the relation of the four dimensional Planck mass is expressed with five dimensional Planck mass as

$$M_{(4)}^2 = 2y_2 M_{(5)}^3. \quad (23)$$

Therefore, the four dimensional Planck mass is relevant to five dimensional Planck mass and the position of brane.

From Eq. (22), on the second brane i.e. $y = y_2 \neq 0$ we have

$$\frac{A_2 \dot{\nu}}{A_2 \nu} = 2. \quad (24)$$

So, the relation of ν and A_2 is

$$\nu = C A_2^2, \quad (25)$$

where C is a integral constant. We can eliminate the arbitrary function ν in Eq. (5). Therefore, the (5) is written as

$$A_2^2 = (\mu^2 + k) y_2^2 - 2CA_2^2 |y_2| + \frac{C^2 A_2^4 + K}{\mu^2 + k}. \quad (26)$$

And this equation is rewritten as

$$\left[A_2^2 - \frac{1}{2C^2} (2C|y_2| + 1)(\mu^2 + k) \right]^2 = \frac{1}{C^4} (|y_2| + \frac{1}{4})(\mu^2 + k)^2 - \frac{K}{C^2}. \quad (27)$$

Therefore, the solutions of this equation are

$$A_2^2 = \frac{1}{2C^2} (2C|y_2| + 1)(\mu^2 + k) \pm \sqrt{\frac{1}{C^4} (|y_2| + \frac{1}{4})(\mu^2 + k)^2 - \frac{K}{C^2}}, \quad (28)$$

where for $A_2^2 \geq \frac{1}{2C^2} (2C|y_2| + 1)(\mu^2 + k)$, “+” sign is taken; while for $A_2^2 \leq \frac{1}{2C^2} (2C|y_2| + 1)(\mu^2 + k)$, we choose “-” sign. This scale factor gives a clearer geometrical description about the 5D spacetime. For $K = 0$, we can find $I_1 = I_2 = I_3 = 0$ and 5D bulk is a 5-dimensional flat spacetime. So, the scale factor in the 5D flat spacetime is

$$A_2^2 = \frac{1}{C^2} \left(C|y_2| + \frac{1}{2} \pm \sqrt{|y_2| + \frac{1}{4}} \right) (\mu^2 + k). \quad (29)$$

For a flat 3D space i.e. $k = 0$, in the 5D flat spacetime, Eq. (29) is simplified as

$$A_2^2 = \frac{1}{C^2} \left(C|y_2| + \frac{1}{2} \pm \sqrt{|y_2| + \frac{1}{4}} \right) \mu^2. \quad (30)$$

Using the red-shift relation

$$A_2 = \frac{A_{20}}{1+z}, \quad (31)$$

from the Eq.(30), we have

$$\frac{1}{C^2} \left(C|y_2| + \frac{1}{2} \pm \sqrt{|y_2| + \frac{1}{4}} \right) \mu^2 = \frac{A_{20}^2}{(1+z)^2}. \quad (32)$$

So, for different y_2 , the A_{20} is different. In other words, we obtain different A_{20} in different brane. Therefore, if considering when $z = 0$, $A_{20} = 1$ on different brane, the redshift z should be redefined on different brane.

In fact, these two solutions all can induce the Λ CDM universe on $y \neq 0$ brane. C is an arbitrary constant. Substituting $\nu = CA_2^2$ into Eq. (19), the deceleration parameters is rewritten as

$$q_2 = \frac{1}{2} \left[-\frac{3C(1 - \Omega_{20})}{1 + CA_2(1 - \Omega_{20})} + 1 \right]. \quad (33)$$

The present value of deceleration parameters q_2 is

$$q_{20} = \frac{1}{2} \left[-\frac{3C(1 - \Omega_{20})}{1 + C(1 - \Omega_{20})} + 1 \right]. \quad (34)$$

6 *Y. Ping et al.*

Our universe is accelerating, so the deceleration parameter is $q_{20} < 0$. Therefore, the range of C is $C > 1/(2 - 2\Omega_{20})$ or $C < -1/(1 - \Omega_{20})$. Adopting $q_{20} = -0.5$ and $\Omega_{20} = 0.3$, we have $C = 20/7 \approx 3$.

If we utilize the result $\nu = CA_2^2$ on $y = 0$ brane i.e $\nu = CA_1^2$, the density on $y = 0$ brane will be $\rho_1 = 6C/\kappa_{(5)}^2$ and $p_1 = -6C/\kappa_{(5)}^2$. Therefore, the $y = 0$ brane is dominated by abnormal matter with $w = -1$.

4. Conclusions

In this paper, the exact global solutions of brane universes are discussed. The solutions contain two arbitrary functions μ and ν of time t . On the brane, the Friedmann equations are modified by the extra term with ν . Therefore, the arbitrary function ν will influence the evolution of our universe. Then we find dimensionless density parameters and deceleration parameters are relate to the arbitrary function ν . Λ CDM universe is the most simple model and not ruled out by present astronomical observation. But we do not know where it comes from. Now, in order to derive Λ CDM universe from the $5D$ spacetime, the arbitrary function ν is eliminate in the scale factor of global brane. So the more exact solutions of the global brane are obtained. Then a clear $5D$ manifold is presented.

Acknowledgments

This work was supported by NSF (10573003), NSF (10647110), NSF (10703001), NBRP (2003CB716300) of P. R. China and DUT 893321.

References

1. A. G. Riess et al., *Astrophys. J.* **116** 1009 (1998) astro-ph/9805201.
2. S. Perlmutter, et.al., *Astrophys. J.* **517** 565 (1999), astro-ph/9812133.
3. A. Einstein, *Sitz. Preuss. Akad. d. Wiss. Phys.-Math* **142** 87 (1917).
4. A. Friedmann, *Z. Phys.* **10** 377 (1922).
5. E.P. Hubble, *Proc. Natl. Acad. Sci.* **15** 168 (1929).
6. S. Weinberg, *Rev. Mod. Phys.* **61** 1 (1989).
7. V. Sahni and A.A. Starobinsky, *Int. J. Mod. Phys. D* **9** 373 (2000), astro-ph/9904398.
8. Varun Sahni, *Class.Quant. Grav.* **19** 3435 (2002)astro-ph/0202076.
9. V. Sahni and A.A. Starobinsky, *Int. J. Mod. Phys. D* **15** 2105 (2006), astro-ph/0610026.
10. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* **429** 263 (1998), hep-ph/9803315.
11. N. Arkani-Hamed, S.Dimopoulos, G. Dvali, *Phys. Rev. D* **59** 086004 (1999), hep-ph/9807344.
12. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* **436** 257 (1998), hep-ph/9804398.
13. P. Horava, E. Witten, *Nucl. Phys. B* **460** 506 (1996), hep-th/ 9510209.
14. E. Witten, *Nucl. Phys. B* **471** 135 (1996), hep-th/9602070.
15. P. Horava, E. Witten, *Nucl. Phys. B* **475** 94 (1996), hep-th/ 9603142.
16. L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83** 3370 (1999), hep-ph/ 9905221.
17. L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83** 4690 (1999), hep-th/ 9906064.

18. P. Binetruy, C. Deffayet, D. Langlois, *Nucl. Phys. B* **565** 269 (2000), hep-th/9905012.
19. Roy Maartens, *Living Rev. Rel.* **7** 7 (2004), gr-qc/0312059.
20. Philippe Brax, Carsten Van De Bruck and Anne-Christine Davis, *Rep. Prog. Phys.* **67** 2183C2231 (2004).
21. V Sahni and Y Shtanov, *Journal of Cosmology and Astroparticle Physics*, **11** 014 (2003).
22. C. Deffayet, G. Dvali, G. Gabadadze, *Phys. Rev. D* **65** 044023 (2002).
23. H. Y. Liu and B. Mashhoon, *Ann. Phys. (Leipzig)* **4** 565 (1995).
24. H. Y. Liu and P. S. Wesson, *Astrophys. J.* **562** 1 (2001), gr-qc/0107093.
25. L. X. Xu, H. Y. Liu and B. L. Wang, *Chin. Phys. Lett.* **20** 995 (2003), gr-qc/0304049.
26. B. L. Wang, H. Y. Liu and L. X. Xu, *Mod. Phys. Lett. A* **19** 449 (2004), gr-qc/0304093.
27. Lixin Xu and Hongya Liu, *Int. J. Mod. Phys. D* **14** 883 (2005), astro-ph/0412241.
28. B. R. Chang et al, *Mod. Phys. Lett. A* **20** 923 (2005), astro-ph/0405084.
29. H. Y. Liu et al, *Mod. Phys. Lett. A* **20** 1973 (2005), gr-qc/0504021.
30. L. X. Xu, H. Y. Liu and C. W. Zhang, *Int. J. Mod. Phys. D* **15** 215 (2006), astro-ph/0510673.
31. C. Zhang et. al., *Mod. Phys. Lett. A* **21** 571 (2006), astro-ph/0602414.
32. Yongli Ping et. al. *Int. J. Mod. Phys. A* **22**, 985 (2007) gr-qc/0610094.
33. Xinhe Meng, Jie Ren, astro-ph/0701298.
34. Sanjeev S. Seahra, *Phys. Rev. D* **68**, 104027 (2003), hep-th/0309081.
35. J. Ponce de Leon, *Mod. Phys. Lett. A* **16**, 2291 (2001), gr-qc/0111011.
36. H. Y. Liu, *Phys. Lett. B* **560** 149 (2003), hep-th/0206198.
37. Y. L. Ping et. al., *Int. J. Mod. Phys. D* **16**, 1633 (2007), arXiv:0707.2825.